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# PHASE NOISE LIMITATION DUE TO AMPLITUDE FREQUENCY EFFECTS IN STATE-OF-THE-ART QUARTZ OSCILLATORS

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#### **ABSTRACT**

During the past two decades very important advances have been accomplished in reducing the phase modulation (PM) noise in state-of-the-art bulk wave quartz crystal oscillator. Various limitations have been significantly reduced, especially those related to dynamic temperature fluctuations, temperature gradients, 1/f noise in the electronic and amplitude frequency effects.

Amplitude frequency effects have been studied in the past [1] because they are so large in AT-cut resonators. The introduction of the SC-cut significantly reduced the sensitivity to amplitude fluctuations as well as the sensitivity to temperature fluctuations. The amplitude frequency effect in modern SC-cut resonators at 5 or 10 MHz is, however, far from negligible and its importance relative to temperature effects and 1/f amplifier noise must now be reexamined to chart the path to further advances in short-term stability.

In this paper we show comparisons of the amplitude frequency effect in traditional 5 MHz AT-cut resonators to our results obtained with 5 MHz BVA AT-cut, 5 MHz BVA SC-cut, 10 MHz BVA AT-cut and several designs of 10 MHz BVA SC-cut resonators. We also compare these measurements with those obtained from 100 MHz resonators.

A simple model in which an additional noise source arising from the amplitude frequency effect is introduced in the input of the resonator excited at a given power. This permits us to estimate the contribution of the amplitude frequency effect to  $\sigma_{V}(\tau)$ .

**Key Words**: Amplitude frequency effect, phase noise, short term stability, amplitude noise, BVA quartz resonator, quartz oscillator.

### 1. AMPLITUDE FREQUENCY EFFECTS

So as to investigate theoretically the frequency amplitude effect, simple conditions can be assumed [1]. Indeed, it is not necessary to calculate the resonant frequency of the resonator from its dimensions, only the main deformation need be considered in an unidimensional model. Therefore a plane wave

propagation will be considered in a infinite single-rotated plate. In this simplified model, the electric field will be perpendicular to the plane wave and it will depend on both time and space variable y.

In this case, the amplitude frequency effect can be deduced:

$$\frac{\Delta f}{f_0} = \frac{f - f_0}{f_0} = \frac{3 n^2}{256} \cdot \frac{c_{666}^D}{c_{66}^D} \cdot \frac{\pi^2 I_0^2}{e_{26}^2 S^2 \omega_0^2}$$
$$= A \cdot \left(\frac{n}{f_0}\right)^2 \cdot \frac{I_0^2}{S^2} \qquad (Eq. 1)$$

where S is the plated surface, n the overtone rank and  $I_0$ , the current amplitude which was expressed in terms of mechanical vibration components.  $c_{66}^{\rm p}$  and  $c_{666}^{\rm p}$  are the elastic coefficients of the second and fourth order defined at constant electric displacement for the considered single rotated cut,  $e_{26}$  being the piezoelectric coefficient. For a given cut, A is a constant.

This expression exhibits a quadratic variation law for frequency difference as a function of **current** amplitude. Furthermore, the frequency shift is proportional to the square of the overtone rank divided by the resonant frequency and inversely proportional to the square of the electrodes area.

Considered alone, this expression would suggest that crystal should be driven at low current for better frequency stability.

For frequency and time applications, it seems better not to consider the crystal current (which depends strongly to the energy trapping inside the resonator bulk) but the crystal power (P) which includes the motional resistance  $(R_m)$  of the resonator. So, the expression can be written as follows:

$$\frac{\Delta f}{f_0} = A \cdot \left(\frac{n}{f_0}\right)^2 \cdot \frac{P}{R_m \cdot S^2}$$
 (Eq. 2)

Although the constant A does not have the same form for single and double rotated cuts, the previous expression can nevertheless help us to make some comparisons between different kinds of resonators with various electrodes and different overtones. For instance, for a given resonator, the ratio between the amplitude frequency effects of two different overtones is just in the inverse ratio of their motional resistance. In the following, we will use this expression to discuss our measurements.

# 2. MEASUREMENTS OF AMPLITUDE FREQUENCY EFFECTS ON SEVERAL 5, 10 AND 100 MHz RESONATORS

The measurements of the 5 and 10 MHz have been performed using a resonator bridge system (according to IEC standard). Furthermore, they have been put in an oven stabilized at a temperature close to their turnover points (about 80°C). Using computer, we check the phase difference between the input and the output levels on a "PI" bridge in which the quartz resonator is inserted. So, for a given signal amplitude delivered by the synthetizer, we store the frequency for which the phase difference is zero.

When the drive level is important, the amplitude frequency effect arises and jump phenomena may appear on resonance curve as illustrated in Fig. 1. This curve, drawn for a 5MHz AT-cut regular unit driven at 350µW, displays an hysteresis effect around the zero-phase of about 6 Hz.

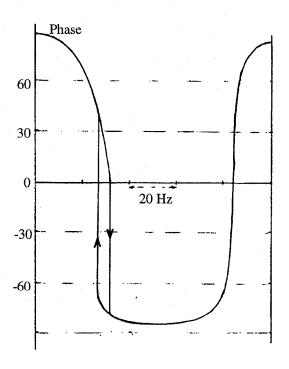
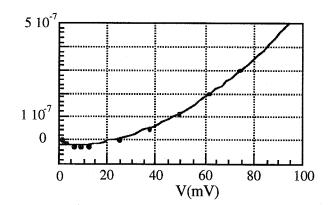


Fig. 1: Phase-frequency curve around the resonance of the AT-cut 5MHz resonator

For the previous resonator, we have measured the fractional frequency shift versus either crystal voltage or crystal drive level between 0.02 and 500  $\mu W$ . The quasiperfect quadratic variation law exhibits a minimum at about 10 mV or 100  $\mu A$  through the resonator. From this result it can be deduced that, when a crystal operates around this minimum, the fluctuation effect on frequency can be strongly reduced. Unfortunately, the corresponding crystal power is too low for most applications (1  $\mu W$ ) (see Fig. 2).



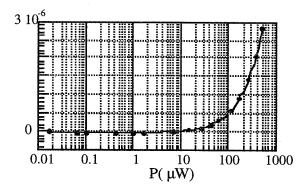


Fig. 2: "Classical" 5MHz AT-cut resonator: Fractional Frequency shift vs Crystal Voltage and Crystal Power

Fig. 3 illustrates frequency shifts versus crystal voltage for two BVA 10MHz SC-cut resonator. The resonator which exhibits the lowest shift works on the 5th overtone and has an electrode diameter of 5.6 mm. This is compared to a 3rd overtone Xtal with an electrode diameter of 4.5 mm. Furthermore, the ratio of their motional resistances is equal to 2 (5th Ov. / 3rd Ov.). So, we can see that the measurements are in agreement to the previous theoretical expression for the amplitude frequency effect found in Eq. 2.

## BVA SC-cut 10MHz (3rd and 5th Ov.)

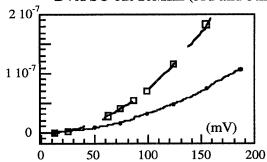


Fig. 3: BVA 10MHz SC-cut 3rd and 5th Ov. resonators: Fractional Frequency shift vs
Crystal Voltage

Finally, Fig. 4 exhibits various fractional frequency shifts versus drive level. Furthermore, the following table (Table 1) presents the slope of these linear curves and the main characteristics of the measured resonators. As can be predicted, the ratio of the slope between 3rd and 5th overtones of the first "classical" AT-cut resonator is in accordance with the theoretical calculation of Eq. 2.

Regular 5 MHz AT-cut units present almost the same type of variations by respect to motional resistance and diameter of electrodes as the BVA resonators. However, the amplitude frequency effect is two times lower for the BVA than for the "classical" 5 MHz AT-cut.

From this table, we can also deduce that the frequency shift due to amplitude frequency effect for the 5th overtone 10 MHz BVA SC-cut is, with the same crystal power, slightly lower than this effect for the 3rd Ov. 10 MHz BVA SC-cut.

As for the 100 MHz SC-cut resonators, the relations between the frequency-amplitude effect and the drive level are similar, but the ratio between the diameters of electrodes being equal to 2, it is not possible to apply the equation (2).

In fact, since the displacement amplitude of the vibration is not homogeneous under the electrodes, we have to introduce a *corrective factor* to take into account the energy trapping and, so, the real diameter of the vibrating area.

Table 1: Slope of the relation between  $\Delta f/f$  and the drive level for various resonators.

Resonator type	Slope (in 10 <sup>-9</sup> )	Overtone rank	Real diam. (mm)	R <sub>m</sub>	A (*10 <sup>-4</sup> ) /μW
AT 4.9MHz AT 8.19MHz	10.1 5.4	3rd 5th	9	55 100	0.60 0.59
Regular AT 5MHz	5.5	5th	11	95	0.48
BVA AT 5MHz	2.7	5th	10	80	0.13
BVA AT 10MHz	3.62	5th	6.5	90	0.14
BVA SC 10MHz	0.77	3rd	4.5	100	0.021
BVA SC 10MHz	0.62	5th	5.6	200	0.03
BVA SC 5MHz	0.48	3rd	10	75	0.062
SC 100MHz Standard	0.29	5th	3.08	75	0.049
SC 100MHz Large Electrodes	0.66	5th	4.32	60	0.34
SC 100MHz Small Electrodes	0.67	5th	2.16	110	0.04
BT-cut 10MHz	- 0.1	Fund	3.5	45	0.016

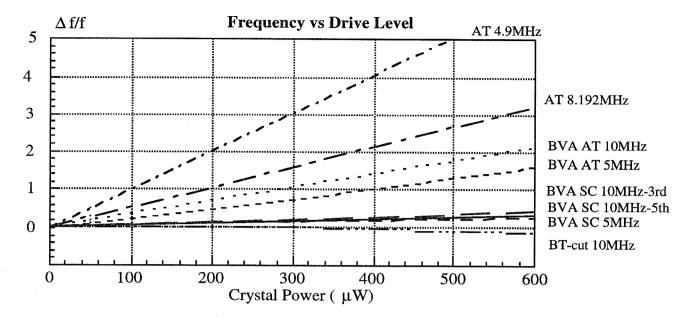


Fig. 4: Fractional frequency change (in ppm) vs crystal power (in  $\mu W$ )

# 3. AMPLITUDE FREQUENCY EFFECT CONTRIBUTION TO $\sigma_y(\tau)$ AND CONCLUSION

A simple theoretical model has been developed. In this model, an additionnal noise source arising from amplitude frequency effects is introduced to feed the resonator working at a given drive level. The program uses more or less regular software to determine in  $\sigma_y(\tau)$  the contribution of amplitude frequency effects noise, i.e. fractional frequency fluctuations arising from amplitude noise (Fig. 5).

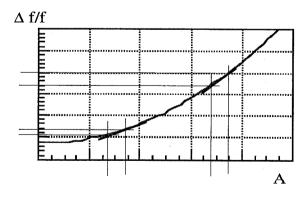


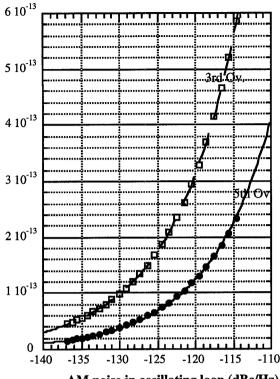
Fig. 5: Fractional frquency change due to amplitude frequency effect

Table 2 shows the excess flicker frequency noise that would be observed in an oscillator, expressed in terms of fractional frequency, that arises from the amplitude frequency effect in the resonator and AM noise, with the usual 1/f spectrum relative to the carrier signal in a 1 Hz bandwith (dBc/Hz). For this table we assume a crystal drive power of  $100~\mu W$  and show the effect for three different 1 Hz intercepts of AM noise. Tables 1 and 2 can be used to show the results for any combination of AM noise and spectrum as well as crystal power. The values chosen for Table 2 correspond to the range of experimental values commonly encountred (see also Fig. 6).

Table 2: Excess flicker noise due to the amplitude freq. effect in various resonators for 1/f AM noise with the specified 1 Hz intercept

Δf/f (in 10 <sup>-13</sup> )	-135 dBc/Hz	-125 dBc/Hz	-115 dBc/Hz
5MHz AT-cut "Bliley"	2.2	6.5	21
5 MHz BVA AT-cut	0.85	2.6	8.3
10 MHz BVA SC-cut 3rd overtone	0.4	1.2	4.1
10 MHz BVA SC-cut 5th overtone	0.16	0.5	1.5
5 MHz SC-cut	0.20	0.54	1.6
100 MHz SC-cut Large Electrodes	0.35	1.1	3.4
100 MHz SC-cut Small Electrodes	0.22	0.72	2.2

### BVA 10 MHz SC-cut (3rd and 5th Ov.)



AM noise in oscillating loop (dBc/Hz)

Fig. 6: Example of excess flicker noise due to the amplitude frequency effect and AM noise in oscillating loop

From the results of Table 2, it is clear that the influence of the amplitude frequency effect is not totally negligible and varies significantly from one Xtal type to another. It is also clear, however, that the amplitude frequency effect is presently not the limiting factor for  $\sigma_y(\tau)$ , when the resonator is adequately chosen and carefull attention is paid to amplitude noise in the oscillating loop.

#### REFERENCES

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